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Now put $u'=u$, $v'=v$, $w'=w$; then (6) becomes $(u+\alpha v+\alpha^2 w)^2(u+\beta v+\beta^2 w)^2 \times(u+\gamma v+\gamma^2 w)^2$, where

$$U=u^2+2avw+amw^2 \ldots.(A),$$
$$V=2uv-2nvw-(mn-a)w^2 \ldots.(B),$$
$$W=2uw+v^2+2mvw+(m^2-n)w^2 \ldots.(C).$$

Whence we have the general solution of

$$U^3+m\,U^2V+(m^2-2n)U^2W+nUV^2+(mn-3a)UVW+(n^2-2am)UW^2+aV^3 +amV^2W+anVW^2+a^2W^3=z^2 \ldots.(7).$$

To make (7) applicable to (1), we set $U=x$, $V=y$, $W=0$. Then from (A), (B), and (C), we find

$$u=\frac{v^2+2mvw+(m^2-n)w^2}{2w},$$
$$x=u^2+2avw+amw^2,$$
$$y=2uv-2nvw-(mn-a)w^2,$$

in which v and w may be chosen at pleasure. When $m=n=0$,

$$u=-\frac{v^2}{2w^2},\quad x=u^2+2avw,\quad y=2uv+aw^2,$$

Hence, since u and v may be chosen at pleasure, there is an infinite number of solutions corresponding to each integral value of a.

Excellent solutions were also received from *G. B. M. ZERR*, and the *PROPOSER*.

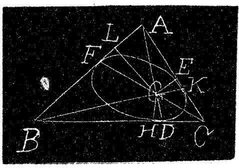
AVERAGE AND PROBABILITY.

126. Proposed by J. SCHEFFER. A. M.. Hagerstown. Mo.

Find the average ellipse inscribed in a triangle, so that the sides of the triangle are tangent to the ellipse.

Solution by G. B. M. ZERR. A. M.. Ph. D.. Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let ABC be the triangle; DEF the inscribed ellipse; D, E, F the points where the sides touch the ellipse; G, the point of concurrence of AD, BE, CF; $GH=l$, $GK=m$, $GL=n$; l, m, n, the perpendiculars from G on the sides; BA, BC, the axes of coördinates; a, b, c the sides of the triangle; $\triangle$, its area; P, the area of the ellipse; and Q, the average area required. Then



$$[(blm+cln)x+(aln+bmn)y-acln]^2-4b^2lm^2nxy=0,$$

is the equation to the ellipse.

$$\therefore P=\frac{\pi a^2bc^2lmn\sin B}{2\sqrt{[(ablm+acln+bcmn)^3]}}=\frac{\pi abc\triangle lmn}{\sqrt{[(ablm+acln+bcmn)^3]}}.$$

But $al+bm+cn=2\triangle$.

$$\therefore P=\frac{\pi\triangle acln[2\triangle-al-cn]}{\sqrt{[(2\triangle al+2\triangle cn-acln-a^2l^2-c^2n^2)^3]}}.$$

From the equation to the ellipse, we get

$$BD=u=\frac{acn}{bm+cn}=\frac{acn}{2\triangle-al},\quad BF=v=\frac{acl}{al+bm}=\frac{acl}{2\triangle-cn}.$$

$$\therefore al=\frac{2\triangle v(a-u)}{ac-uv},\quad cn=\frac{2\triangle u(c-v)}{ac-uv}.$$

$$\therefore P=\frac{\pi\triangle uv\sqrt{[(a-u)(c-v)]}}{\sqrt{[(av+cu-uv)^3]}}.$$ Let $a-u=t$, $c-v=z$.

$$\therefore P=\frac{\pi\triangle[a-t][c-z]\sqrt{[tz]}}{\sqrt{[(ac-tz)^3]}}.$$

$$\therefore Q=\int_0^a\int_0^c Pdtdz\Big/\int_0^a\int_0^c dtdz=\frac{\pi\triangle}{ac}\int_0^a\int_0^c\frac{(a-t)(c-z)\sqrt{(tz)}dtdz}{\sqrt{[(ac-tz)^3]}}.$$

Let $tz=ac\sin^2\theta$, $\theta'=\sin^{-1}\sqrt{(t/a)}$.

$$\therefore Q=\frac{2\pi\triangle}{a}\int_0^a\int_0^{\theta'}\frac{(a-t)(t-a\sin^2\theta)\sin^2\theta dtd\theta}{t^2\cos^2\theta}$$

$$=\frac{\pi\triangle}{a}\int_0^a\left(\frac{a-t}{t^2}\right)\{3a\sin^{-1}\sqrt{(t/a)}-2t\sin^{-1}\sqrt{(t/a)}-3\sqrt{[t(a-t)]}\}dt.$$

Let $t=a\sin^2\varphi$.

$$\therefore Q=2\pi\triangle\int_0^{\frac{1}{2}\pi}(3\varphi-2\varphi\sin^2\varphi-3\sin\varphi\cos\varphi)\cot^3\varphi d\varphi=\tfrac{1}{2}\pi^2\triangle(7-10\log^2)$$

127. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

What is the probable error of the volume of a rectangular parallelopiped whose edges measured by the repeated application of a unit of measure are found to be a, b, c, supposing that the probable error of a line so measured whose length is found to be l is $r\sqrt{l}$?